STUDY OF MATHEMATICAL MODELLING FOR HEAT TRANSFER IN FINS

SYNOPSIS

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in
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By

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1. Title: STUDY OF MATHEMATICAL MODELLING FOR HEAT TRANSFER IN FINS

2. Introduction

Firstly, Harper and Brown (1923) gave analytical theory of heat. Till 18^{th} century, heat was regarded as a fluid known as calorie. Just thereafter, Benjamin Thompson, who later became Count Rumford, rejected this theory of heat. He concluded that heat is not a fluid but it is a form of energy. Heat transfer (HT) refers to the exchange of energy in the form of heat (Zemansky and Dittman 1998). The term HT refers to describe the moment of energy from one object to another in the form of heat. When a hot object touches a cooler body, the heat of the hotter body goes consistently toward the cooler object until the temperatures of both objects are equal. HT produces the sensation of hotness and coldness to us.

Heat always moves from hot to cold temperature. The key mechanisms of HT process is classified in three categories i.e., conduction, convection and radiation (Zemansky and Dittman 1998). Conduction HT occurs only in solids. Whenever two layers of a solid object are held at distinct degrees of warmth, then heat will proceed to move through the hotter layer over to the colder layer caused by a vibrating moment of the particles. For example, heating metal strip. Convection HT occurs in liquid or gas. It is the flow of liquid or gas that transfer heat from high to lower temperature region, because of the actual movement of molecules. For example, boiling water in a pan and for HT through radiation any medium is not required. Radiation is another way of HT via radio waves. For example, HT through sun and materials are heated through microwave.

During mechanical processes heat is generated inside the machines. The primary issue here is how to effectively transmit this heat towards the atmosphere around us. If the heat will not dissipated towards the surrounding medium, this could raise the temperature of the entire apparatus. That may creates an instance of overheating and breakdown of the system. For the purpose of tackling such a problem, fins are receiving popularity as tools for transferring heat toward the surroundings due to their basic layout

and versatility for handling coupled heat transmission. Fins are also recognized as extended surfaces that are used to rise the surface area to speed up the rate of HT within the hot surface and the surrounding medium. The temperature, where fin is connected to the heated base is higher and gradually drops over fin length. Firstly, Harper and Brown (1923) gave mathematical analysis for extended surfaces. That surface was initially addressed as a cooling fin, which ended up being simply a fin. Jakob (1949) found that the mathematical study of fins can be dated back in early seventies. During that era, studies are undertaken to determine the heat conductivity of numerous materials. Wax-coated rods of several metals were heated to demonstrate the corresponding melting patterns. Later a mathematical analyses of the thermal changes in these rods were examined by Fourier (1878).

These days, expanded surfaces are constructed from metallic alloys known for their excellent thermal characteristics. Fins are typically made from aluminum, stainless steel, or copper. The shape of the fins play a significant role in optimizing fin performance (Kundu and Das 2002). It is usually understood that whenever the length of a fin raised, the HT rate generated by the fin drops, and the whole fin surface might not be utilized to its maximum capacity. As a result, designers are perpetually seeking to find the best fin that may minimize the contribution of fin material to HT or improve the HT rate over certain fin regions. An examination of the fin's design and dimensions may suggest the smallest quantity of materials needed to make a fin for the described HT. Kern (1972) examine the fins design dimension which satisfy the provided optimization requirements. There are numerous fin forms i.e., rectangular, annular, elliptic, parabolic, pin fins, etc., can be used to dissipate heat into the surroundings (Kraus et al. 2001). The applications of fins are increasing in a broad spectrum of industries, i.e. air conditioning, refrigeration, internal combustion engines, and so on (Zhang et al. 2013).

Every element possessing porosity is known as a permeable materials. In several scenarios, HT occurs through permeable fins, that are composed of fluid-solid mixes. Permeable fin are lightweight, having low maintenance, low thermal conductivity and high HT rate in comparison with solid one (Hundu et al. 2012). Nowadays, porous fins with the addition of different shapes of nano-fluid particles are widely used for heat transmission (Hosseinzadeh at al. 2022).

Bearing in mind the foregoing facts, the current research entitled "Study of mathematical modelling for HT in fins" has been proposed with the following objectives

1. A comparative analysis of different numerical methods in porous fin with temperature-dependent thermal conductivity and internal heat generation

2. Study of HT in porous fin with temperature-dependent internal heat generation, also considering the space and temperature-dependent thermal conductivity

3. Investigation of a new hybrid numerical technique to solve the HT in 2-D elliptic and plate fins in

study and unsteady state experiencing a range of conditions, including dry, wet and partially wet, subjected to the most generalized boundaries

4. Investigation of a mathematical model that deals with HT in 2-D square fin via convection and radiation with temperature-dependent HT coefficient and variable surface emissivity
5. Study of HT in a stretching, stagnant and shrinking longitudinal fin with an exponential profile
6. Study of HT in fully wet moving porous fin subjected to nanofluid with spherical, needle and disk types of the nanofluid particles

The rational of the proposed study

Several investigators adopted several approaches to explore various concerns associated to distinct layouts of fins. The focus of the research aims to establish a new hybrid mathematical approach along with the comparative investigation of distinct approaches, and illustrate its implementation within a one or two-dimension non-linear or linear system incorporating most generalized borders. This addresses the field of application gap for the hybrid methodology.

3. Expected new knowledge

- Applicability of different numerical methods and effect of basis functions on HT in porous fin.
- Investigation of a new hybrid numerical method and its application in the solution of linear and non-linear problems.
- The effect of radiation and variable emissivity in 2-D square fin.
- Knowledge about enhancement, the cooling effect of the fin with variable surface emissivity.
- To find best shape of nanoparticle, which gives the better HT rates.

4. Review of literature

Nowadays, fins are receiving popularity as tools for transferring heat toward the surroundings due to their basic layout and versatility for handling coupled heat transmission. Fins are also recognized as extended surfaces that are used to increase surface area to speed up the rate of HT within the hot surface and the surrounding medium. These surfaces are constructed from metallic alloys known for their excellent thermal characteristics. Fins are typically made from aluminum, stainless steel, or copper. There are numerous fin forms i.e., rectangular, annular, elliptic, parabolic, pin fins, and so on can be used to dissipate heat into the surroundings. The applications of fins are increasing in a broad spectrum of

industries, i.e. air conditioning, refrigeration, internal combustion engines and so on. According to the design of the fin a new mathematical models can be developed, boost overall fin quality, optimize matter prices, and preserve fin effectiveness. A brief overviews of this kind of problem has been examined by multiple authors, which are summarized below:

Kramer and Stockman (1963) examine 1-D heat flow within a fin-tube configuration. This configuration involves surfaces with changing emissivity and thermal properties that are influenced by temperature variation. Campo and Wolko (1973) investigate the impacts of thermal properties that are contingent on temperature and emissivity on the surface of fin with an addition of power law kind of heat transfer coefficient. Kraus et al. (2001) gave detailed idea of expandable structures in their book. The application of different types of fins used to increase the HT within surface and fluid has been discussed in detail. The concept of fin efficiency is also covered in this book. Chiu (2002) applied the adomian decomposition approach to derive the HT in fin. Additionally, productivity of the fin in cases where thermal conductivity exhibited a linear dependence on temperature was calculated. This method yields an analytical solution, which is represented as an infinite power series. Lin and Jang (2002) utilized the conventional 1-D sector technique to examine the usefulness of an 2-D dry, moderately wet, and totally wet elliptic fin.

Cengel (2006) discussed heat and mass transmission as well as its applications to a variety of contexts, including industrial operations, home appliances, technological gadgets, food processing, and commercial structures. Also, studied physics and physical reasons to cultivate an intuitive grasp of heat transfer. Kiwan (2007) studied temperature profile in porous fin, demonstrating that the fin outperforms its solid counterpart because of its increased ratio of surface area to volume ratio as compared to porous one. Seyf and Layeghi (2010) examine HT rate in elliptic pin fin by implementing a finite volume technique. Azarkish et al. (2010) employed a genetic algorithm to investigate the optimal geometry and fin count for maximizing the amount of HT via conduction-radiation in a 1-D context. Bergman et al. (2011) explained the text discusses the concepts of mass and energy movement, as well as the connection among thermodynamic and HT. Further, they redesigned 1-D equilibrium state transmission, with new substance now covering conductivity in permeable media, thermal energy production, and nano-scale phenomena.

Kundu and Wongwises (2012) utilized he Adomian decomposition approach to examine the temperature profile in a rectangular plate fins exposed to convective and radiative conditions. Their investigation also considered variations in the thermal properties of the fins. Aziz and Torabi (2012) conducted a study to examine a convective-radiative fin characterized by the properties that varies with temperature. For the solution of this problem, they employed both the fourth and fifth-order Runge-Kutta-Fehlberg methods. Kundu et al. (2012) examine HT and best shape of a porous fin. They utilized adomian de-

composition approach for solution. The study findings indicated that HT rates are notably higher in cases with exponential profiles featuring negative power factors when contrasted with rectangular profiles. Furthermore, they demonstrated that the fin's performance excelled when employing exponential profiles characterized by positive power factors in comparison to other profile shapes. Hatami and Ganji (2013) examine HT rates specifically in the context of circular porous fins. The Runge-Kutta 4^{th} and the Least Square techniques was applied for the solution. Their analysis led to the conclusion that the results obtained through the Least Square Method exhibited a notable degree of agreement. Singh et al. (2013) utilize the wavelet collocation technique to review the HT in a continuously movable fin with properties that varies with temperature. They compared the outcomes of Wavelet collocation approach to the exact results and observed a remarkable similarity between the results. Also, they give the study of different parameters on surface heat loss.

Singh et al. (2014) implemented the Legendre wavelet collocation approach for investigating the temperature profiles in convective-radiative fin, considering wavelength-dependent surface emissivity. Hatami and Ganji (2014a) conducted an investigation into the refrigeration efficiency and temperature profile in fully wet circular porous fins. The analysis involved the utilization of both the Least square and the Runge-Kutta 4^{th} order techniques to derive the solution. Bhanja et al. (2014) devised a unique analytical model to assess fin efficiency, providing insights into temperature profile and optimal design specifications for a moving porous fin experiencing heat loss through convection-radiation. They utilized the adomian decomposition approach for solution. Remarkably, the study revealed that the moving porous fin exhibited significantly greater heat dissipation compared to the stationary fin. Hatami and Ganji (2014b) explored longitudinal fin with diverse structures and materials. The least squares technique is employed for the solution. Their findings suggested that exponential fins were advantageous for enhancing HT. Chuan et al. (2015) gave new shape idea for micro-channel heat sinks, wherein traditional solid fins were substituted with porous fins with the aim of reducing pressure drops across the heat sink.

Cuce and Cuce (2015) extensively analyzed the thermal characteristics of straight fins, consider instances for heat conductivity that are steady and relay on temperature. Turkyilmazoglu (2015) conducted an analytical investigation into the stretching/shrinking motion of a fin with a rectangular profile subjected to a convective environment. Their investigation revealed that the shrinking process positively influenced the effectiveness of the fin. Darvishi et al. (2016) employed the pseudo-spectral approach to examine an annular fin with a hyperbolic profile and found that the shorter fin demonstrated higher efficiency in comparison to the longer counterpart. Dononchi and Ganji (2016) studied moving fin with convective-radiative HT. Differential transform technique was applied for HT in rectangular fin. They found that prominence in the heat of fin at the end is proportional to the increased heat generation and conductive-radiative properties. Sobamowo et al. (2017) employed Galerkin approach to analyze the

impact of a naturally convective fin with porous material with factors that relay on temperature.

Turkyilmazoglu (2018) examine HT and effectiveness in the starching fin. The investigators discovered how an growing kind of fin provides the highest fin efficacy. Baslem et al. (2020) examine HT in convective-radiative fully wet longitudinally fin with nanofluid. The Runge-Kutta Fehlberg approach was utilized to derive the outcomes. The research is centered on how geometry and flow-related elements influence the fin's temperature profile. Wang et al. (2021) implemented computational fluid dynamics to evaluate the impacts of vortex generators on an elliptic fin tube's performance across several inclination levels. Bahiraei et al. (2021) execute the finite volume approach to inquire about the dual-phase flow characteristics of a nanofluid incorporating liquid silver over an elliptical pin-fin heat sink. Gireesha et al. (2022) centers on assessing the thermal efficiency of a longitudinally stretched/shrieked fin with an exponential profile, which is completely wet and features a mechanism like conveyor belt for its operation. The solution is achieved through the utilization of the Runge-Kutta-Fehlberg technique. The research concludes that the fusion of a surface shrinking of the fin can ease the adverse impacts of motion and internal heat generation, resulting in enhanced HT rates with in the fin. Wang et al. (2023) utilized the enthalpy porosity technique for researching the solidifying and melting characteristics of a transitional matter inside an elliptical fin.

5. Technical programme

The MATLAB(2015a) program shall be executed to carry out the distinct algorithms and examine the model's variables.

5.1. Model 1:

A comparative analysis of different numerical methods in porous fin with temperature-dependent thermal conductivity and internal heat generation.

5.1.1. Location:

Eternal University, Baru Sahib, Sirmour (HP).

5.1.2. Methodology:

The suggested methodologies are going to be implemented during the investigation of the work.

- **Wavelet collocation method:** The wavelet collocation approach was first introduced by (Razza-

ghi and Yousefi 2000). To get the outcomes of border value problem is expressed as follows:

$$D[v] = f(\xi), \alpha < \xi < \beta, \tag{1}$$

$$v(\alpha) = K, \tag{2}$$

$$v(\beta) = M, \tag{3}$$

here D is the differentiation operator, v and f are true-valued continuous operations of ξ on $L^2(R)$. The probable resolution for Eqs. (1) to (3) appears to be:

$$\tilde{v} = \sum_{j=1}^{n} C_j \omega_j(r), \tag{4}$$

in which $\omega_j(\xi)$, $j = 1,2,\ldots,n$ is a set of wavelet base functions that are perpendicular over $[\alpha,\beta]$ and C_j, $j = 1,2,\ldots,n$ were fixed. In Eq. (1), switching v with $\tilde{v}$ yields

$$R(\xi,C) = D[\tilde{v}] - f(\xi), \tag{5}$$

the Eq. (4) provides $\tilde{v}$ as a probable answer. when inserting the value of $\tilde{v}$ within the Eq. (1), we receive an error. This mistake is referred as residue and indicated by $R(\xi,C)$, precisely stated in Eq. (5). The wavelet-based collocation procedure is implemented to detect the undetermined constant C. From this approach, we choose points $\xi_k, k = 1,2,\ldots,n$ inside the range (α,β) wherein $R(\xi,C)$ eliminates. Such points are identified as collocation points. We consider residual at ξ_k becomes zero. Using this strategy, we discovered a set of non-linear or linear equations based on a mathematical framework that is simple to solve.

- **Finite difference method:** In this technique derivative of different orders replaced by finite approximation. Expanding the function via Taylor series, we might derive expressions for derivatives (Sastry 2012) as below.

  - **Forward difference approximation:**

$$f'(v) = \frac{f(v+d) - f(v)}{d} + O(d).$$

  - **Backward difference approximation**

$$f'(v) = \frac{f(v) - f(v-d)}{W} + O(d).$$

  - **Central difference approximation**

$$f'(v) = \frac{f(v+d) - f(v-d)}{2d} + O(d^2),$$

    and

$$f''(v) = \frac{f(v-d) - 2f(v) + f(v+d)}{d^2} + O(d^2),$$

the grater ordering variables include $O(d)$ and $O(d^2)$. This approach, swap the differential equation's of first or second order derivatives with finite difference approach. Therefore we've got a set equations that are nonlinear or linear will be resolved through prominent technique.

- **Moment method:** In this approach, weight function are treated as a set of polynomial and can be described as

$$W_k(v) = v^k \quad k = 0,1,2,...,m-2,$$

the integral outcomes of the residue and v^k over the operational region V, i.e. i^{th} the moment corresponding to the residue from Eq. (7), yields

$$\int_V R(v)v^k dv = 0 \quad k = 0,1,2,...,m, \tag{6}$$

the Eq.(6) yields a set of equations that are not linear and might be determined via implementing the N-R approach.

5.1.3. Observation to be recorded:

Results will be presented through figures and tables.

5.1.4. Statistical analysis:

Nil.

5.2. Model 2:

Study of HT in porous fin with temperature-dependent internal heat generation, also considering the space and temperature-dependent thermal conductivity.

5.2.1. Location:

Eternal University, Baru Sahib, Sirmour (HP).

5.2.2. Methodology:

The finite difference, Wavelet collection and least square approaches are going to be implemented during the investigation of this work.

- **Least square method:** The premise in this approach is to incorporate residue zero into the given region (Azisik and Ozisik 1993), which is

$$\int_\xi R(\xi)W_j(\xi)d\xi = 0 \quad j = 0,1,2,...,n, \tag{7}$$

the weighted function W_j along with unidentified variables a_j are listed identically within $\tilde{v}$. That yields n mathematical expressions are employed to determine the values of undefined variables a_j. If the permuted average for all squared residue is lowered, then it become feasible to comprehend the significance of this technique. The scalar function corresponding to the residue is shown as follows:

$$S = \int_{\xi} R^2(\xi) d\xi,$$

to identify the lowest possible value associated with S, compute the derivation of S corresponding to each and every anonymous variables should be zero, i.e.

$$\frac{\partial S}{\partial a_j} = 2\int_{\xi} R(\xi)\frac{\partial R}{\partial a_j} d\xi = 0, \tag{8}$$

the weighting function for this approach has been specified as

$$W_j = 2\frac{\partial R}{\partial a_j},$$

the Eq. (8) yields a set of equations that are not linear. The Newton-Raphson technique might be implemented for resolving the resulting equations.

5.2.3. Observation to be recorded:

Results will be presented through figures and tables.

5.2.4. Statistical analysis:

Nil.

5.3. Model 3:

Investigation of a new hybrid numerical technique to solve the HT in 2-D elliptic and plate fins in study and unsteady state experiencing a range of conditions, including dry, wet and partially wet, subjected to the most generalized boundaries.

5.3.1. Location:

Eternal University, Baru Sahib, Sirmour (HP).

5.3.2. Methodology:

The special matrices products, finite difference and Wavelet galerkin approaches are going to be implemented during the investigation of this work.

- **Wavelet galerkin method:** In this technique procedure up-to Eg. (5) is same as mentioned in Wavelet collocation method. After that, we make some changes in these term, where $\omega(\xi)$ is not present and hence it is canceled. Now we obtain a set of equations linear or nonlinear which are easy to resolve via widely known techniques.

- **Special matrices product and its properties:**

  - **Haramard product:** The matrix's $T = (\alpha_{jk})$ and $S = (\beta_{jk})$ possess equivalent order, as determined by (Horn and Johnson 2012) it is represented as $T \circ S$ and described as

$$T \circ S = (\alpha_{jk}) \circ (\beta_{kj}) = (\alpha_{jk}\beta_{kj}).$$

  - **Kronecker product:** The matrices $T = (\alpha_{jk})$ and $S = (\beta_{jk})$ of arbitrary dimension, as introduced by (Henderson et al. 1983) it is expressed as $T \otimes S$ and described as

$$T \otimes S = (\alpha_{jk}S).$$

  - **Khatri-Rao product:** The matrices $T = (\alpha_{jk})$ and $S = (\beta_{jk})$ introduced by (Khatri and Rao 1968) it is expressed as $T \star S$ and described as

$$T \star S = (T_{jk} \otimes S_{jk})_{jk},$$

    here T_{jk} and S_{jk} describes the block matrix.

  - **Face-splitting product:** The matrices $T = (\alpha_{jk})$ and $S = (\beta_{jk})$ of arbitrary dimension, as described by (Esteve et al. 2009) is represented as $T * S$ and described as

$$T * S = (\alpha_{jk}S_j),$$

    the S_j corresponds to the j^{th} row in the matrix S.

5.3.3. Observation to be recorded:

Results will be presented through figures and tables.

5.3.4. Statistical analysis:

Nil.

5.4. Model 4:

Investigation of a mathematical model that deals with HT in 2-D square fin via convection and radiation with temperature-dependent HT coefficient and variable surface emissivity.

5.4.1. Location:

Eternal University, Baru Sahib, Sirmour (HP).

5.4.2. Methodology:

The finite difference approach is going to be implemented during the investigation of this work.

5.4.3. Observation to be recorded:

Results will be presented through figures and tables.

5.4.4. Statistical analysis:

Nil.

5.5. Model 5:

Study of HT in a stretching, stagnant and shrinking longitudinal fin with an exponential profile.

5.5.1. Location:

Eternal University, Baru Sahib, Sirmour (HP).

5.5.2. Methodology:

The Wavelet collocation approach is going to be implemented during the investigation of this work.

5.5.3. Observation to be recorded:

Results will be presented through figures and tables.

5.5.4. Statistical analysis:

Nil.

5.6. Model 6:

Study of HT in fully wet moving porous fin subjected to nanofluid with spherical, needle and disk types of the nanofluid particles.

5.6.1. Location:

Eternal University, Baru Sahib, Sirmour (HP).

5.6.2. Methodology:

The Wavelet collocation approach is going to be implemented during the investigation of this work.

5.6.3. Observation to be recorded:

Results will be presented through figures and tables.

5.6.4. Statistical analysis:

Nil.

6. Collaboration(if any)

Nil.

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Report of the Research Committee on Synopsis of Master's/Ph.D Thesis/Dissertation Research Problem

We have critically examined the synopsis of Thesis/Dissertation Problem entitled:

"Study of mathematical modeling for heat transfer in fins" Submitted BY: **Ms Priti Sharma** Registration No: **BS20PSMA002** for the Ph.D degree in the subject of **Mathematics** to the Akal College of Basic Sciences of the Eternal University, Baru Sahib (HP). It is certified that the research problem in original; programme of work is properly planned, feasible to carry out and is of good academic and practical significance.

Ms Priti Sharma gave a presentation on the above mentioned research problem to the Research Degree Committee and other faculty members and students on 24/05/2024 and his/her performance is Satisfactory/~~Unsatisfactory~~.

Synopsis is recommended for acceptance by the Eternal University.

(Signature, Name and Address of External Expert) Prof. Rakesh Bajaj

Bajaj 24/5/2024

Prof Rakesh Bajaj
Department of Mathematics
JAYPEE University, Waknaghat, Solan (HP)

1. (Dr BS Sohal) TSBamjal
Chairperson, Research Degree Committee

2. Sandipan Gupta
(Member, Research Degree Committee)

3. Dr. Surjan Singh (Guide)
(Member, Research Degree Committee)

4. Dr. Soni Bisht
(Member, Research Degree Committee)

5. Rajeev Thakur
(Member, Research Degree Committee

6. for Dr. S. Upadhyay
(Member, Research Degree Committee)

Recommendation

TSBamjal

(Dean, Postgraduate Studies)

Approved/~~Not approved~~

24/5/24

Vice Chancellor

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Eternal University Baru Sahib (HP)

Certificate of Comprehensive Oral Examination : Ph.D

This is to certify that:

Ms/Mr. **Priti Sharma** Registration No: **BS20PSMA002** the Akal College of Basic Sciences of Eternal University, Baru Sahib (HP) has been examined by us in the subject of **Mathematics**. The oral examination was held on 24/05/2024. Her performance in the oral examination is ~~Satisfactory/Unsatisfactory~~ Satisfactory (To be written by the External Examiner).

Prof. Rakesh Bajaj
(Signature, Name and Address of External Expert)

Bajaj 24/5/24

Prof Rakesh Bajaj
Department of Mathematics
JAYPEE University, Waknaghat, Solan (HP)

1. (Dr BS Sohal) TSBamrah
Chairperson, Research Degree Committee

2. Sandipan Gupta
(Member, Research Degree Committee)

3. Dr. Surjan Singh
(Member, Research Degree Committee)

4. Dr. Soni Bisht
(Member, Research Degree Committee)

5. Rajeev Thakur
(Member, Research Degree Committee

6. for Dr. S. Upadhyay
(Member, Research Degree Committee)

Recommendation

TSBamrah
(Dean, Postgraduate Studies)

Approved/~~Not approved~~

11/6/24
Vice Chancellor

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ETERNAL UNIVERSITY, BARU SAHIB. (H.P.)

Attendance sheet for Synopsis Seminar

Name of student	Registration Number	Signature
MS. PRITI SHARMA	BS20PSMA002	

	Name	Signature
1		
2	Prof R K Bajaj	
3	Mr. DK Ohja	
4	Dr. Sandipan Gupta	
5	Dr. Sujan Singh	
6	Dr. Soni Bisht	
7	Dr. Shivani	
8	Dr. Parvinder Kaur	
9	MS. Harpreet Kaur	
10	Ms. Bharti Thakur	
11	MS. Kamalpreet Kaur	
12	Rajeev Thakur	
13	Priyanka	
14	T. S. Bamal	
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